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**CS 480 Spring 2023 Written Assignment #03**

Due: **Sunday, March 26, 2023 at 11:59 PM CST**

Points: **20**

**Instructions:**

1. Use this document template to report your answers. Name the complete document as follows:

LastName\_FirstName\_CS480\_Written03.docx or pdf

1. Submit the final document to Blackboard Assignments section before the due date. No late submissions will be accepted.

**Objectives:**

1. (5 points) Demonstrate your understanding of First-Order Logic syntax.
2. (7 points) Demonstrate your ability to First-Order Logic sentence into CNF.
3. (8 points) Demonstrate your understanding of conditional probability distributions.

**Problem 1 [5 pts]:**

Convert English sentences to FOL. Write each of the following English sentences using First Order Logic. Use the following predicates and constants only.

* Nationality(**x**, **y**): Predicate. Person **x** nationality is **y**.
* Cousin(**p1**, **p2**): Predicate. Person **p1** is a cousin of person **p2**.
* Drives(**z1**, **z2**): Predicate. Person **z1** drives car brand**z2**.
* American, Dutch, Australian, Swedish, Brazilian: constants denoting some nationalities. This list is not comprehensive. **There are also other nationalities not mentioned here**.
* Chrysler, Ford, BMW: constants denoting some car brands. This list is not comprehensive. **There are also other car brands not mentioned here**.
* Amy, George, Mia: constants denoting some people. This list is not comprehensive. **There are also other people not mentioned in this list**.

1. Mia is either Dutch or Swedish **[1 pts]**. T⊥∨∧≡⇔¬⇒

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| **Your solution:** |
| Nationality(Mia, Dutch) ∨ Nationality(Mia, Swedish) |

1. George is not Brazilian and he drives the same car brand as all his cousins **[1 pts]**.

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| **Your solution:** |
| ¬ Nationality(George, Brazilian) ∧ (∀p Cousin(p, George) → Drives(p,z))  **Note** : p represents Georges cousins and z represents there cars. |

1. Not all people who drive BMWs are Swedish**[1 pts]**.

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| **Your solution:** |
| ¬∀z (Drives(z,BMW) → Nationality(z,Swedish))  **Note** : z represents people. |

1. Every Brazilian has a cousin in Australia **[1 pts]**.

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| **Your solution:** |
| (∀x)(Nationality(x, Brazilian) → (∃y)(Nationality(y, Australian) ∧ Cousin(y, x)))  **Note** : x represents Brazilian people and y represents Australian people. |

1. Driving a Chrysler means that you are either an American or have no cousins **[1 pts]**.

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| **Your solution:** |
| (∀z)(Drives(z, Chrysler) → (Nationality(z, American) ∨ (∀p)(¬Cousin(p,z)))  **Note** : z represents people who drive Chrysler people and z represents American people and p represents cousins of z. |

**Problem 2 [7 pts]:**

Consider the following predicate / First-Order logic sentence:

∀a (∃b (X(a, b) ∧ Y(b)) ⇒ [∀c (¬Z(c, a))])

Assume that X, Y, and Z are predicates. Domains for variables are:

Da = {A1, A2, A3}

Db = {B1, B2}

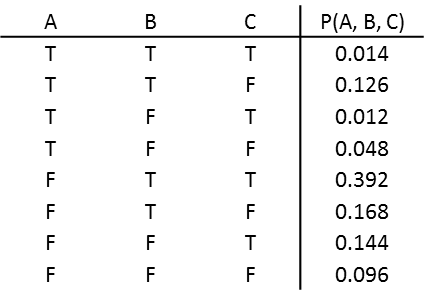
Dc = {C1, C2, C3, C4}

Now convert the sentence above to its **CNF form** (**show and explain all steps**):

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| **Your answer:** |
| **Step 1- Replace ⇒ with equivalent**  ∀a (¬ (∃b (X(a, b) ∧ Y(b)))) ∨ [∀c (¬Z(c, a))])  **Step 2- Reduce ¬ scope of to single literals:**  **Note: Applying Distributive laws when necessary**  ∀a( (¬ ∃b ¬ (X(a, b) ∧ Y(b)))) ∨ ([∀c (¬Z(c, a))])  ∀a ((∀b ¬ (X(a, b)) ∨ ¬ Y(b))))∨ ([∀c (¬Z(c, a))])  ∀a ((∀b ¬ X(a, b) ∨ ¬ Y(b)))) ∨ ([∀c (¬Z(c, a))])  ∀a (( ¬ X(a, b) ∨ ¬ Y(b)))) ∨ ([∀c (¬Z(c, a))])  ∀a ((∀b ( ¬ X(a, b) ∨ ¬ Y(b))))) ∨ ([∀c (¬Z(c, a))])  **Step 3- Drop Universal quantifiers:**  (( ¬ X(a, b) ∨ ¬ Y(b)))) ∨ ([ (¬Z(c, a))])  (( ¬ X(a, b) ∨ ¬ Y(b)))) ∨ ([ (¬Z(c, a))])  ( ¬ X(a, b) ∨ ¬ Y(b) ∨ (¬Z(c, a))  **Step 4- Substituting the Domain Values:**  A1, A2, A3  B1, B2  C1, C2, C3, C4  ***Final CNF:***  ( ¬ X(A1, B1) ∨ ¬ Y(B1) ∨ (¬Z(C1, A1)) ∧ ( ¬ X(A1, B1) ∨ ¬ Y(B1) ∨ (¬Z(C2, A1)) ∧  ( ¬ X(A1, B1) ∨ ¬ Y(B1) ∨ (¬Z(C3, A1)) ∧ ( ¬ X(A1, B1) ∨ ¬ Y(B1) ∨ (¬Z(C4, A1)) ∧  ( ¬ X(A2, B1) ∨ ¬ Y(B1) ∨ (¬Z(C1, A1)) ∧ ( ¬ X(A2, B1) ∨ ¬ Y(B1) ∨ (¬Z(C2, A1)) ∧  ( ¬ X(A2, B1) ∨ ¬ Y(B1) ∨ (¬Z(C3, A1)) ∧ ( ¬ X(A2, B1) ∨ ¬ Y(B1) ∨ (¬Z(C4, A1)) ∧  ( ¬ X(A3, B1) ∨ ¬ Y(B1) ∨ (¬Z(C1, A3)) ∧ ( ¬ X(A3, B1) ∨ ¬ Y(B1) ∨ (¬Z(C1, A3)) ∧  ( ¬ X(A3, B1) ∨ ¬ Y(B1) ∨ (¬Z(C1, A2)) ∧ ( ¬ X(A3, B1) ∨ ¬ Y(B1) ∨ (¬Z(C2, A3))  ∧  ( ¬ X(A1, B2) ∨ ¬ Y(B2) ∨ (¬Z(C1, A1)) ∧ ( ¬ X(A1, B2) ∨ ¬ Y(B2) ∨ (¬Z(C2, A1)) ∧  ( ¬ X(A1, B2) ∨ ¬ Y(B2) ∨ (¬Z(C3, A1)) ∧ ( ¬ X(A1, B2) ∨ ¬ Y(B2) ∨ (¬Z(C4, A1)) ∧  ( ¬ X(A2, B2) ∨ ¬ Y(B2) ∨ (¬Z(C1, A1)) ∧ ( ¬ X(A2, B2) ∨ ¬ Y(B2) ∨ (¬Z(C2, A1)) ∧  ( ¬ X(A2, B2) ∨ ¬ Y(B2) ∨ (¬Z(C3, A1)) ∧ ( ¬ X(A2, B2) ∨ ¬ Y(B2) ∨ (¬Z(C4, A1)) ∧  ( ¬ X(A3, B2) ∨ ¬ Y(B2) ∨ (¬Z(C1, A3)) ∧ ( ¬ X(A3, B2) ∨ ¬ Y(B2) ∨ (¬Z(C1, A3)) ∧  ( ¬ X(A3, B2) ∨ ¬ Y(B2) ∨ (¬Z(C1, A2)) ∧ ( ¬ X(A3, B2) ∨ ¬ Y(B2) ∨ (¬Z(C2, A3)) |

**Problem 3 [8 pts]:**

We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. **Show each probability distributionP**()**as a table/vector**.



1. Probability distribution **P**(B, ¬C)**[2 pts]**

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| **Your solution:** |
| P(B , ¬C) **=** P(B ∧¬C)  This Implies the Probability when B=1 and ¬C=1   |  |  |  | | --- | --- | --- | | Variable | B | ¬B | | C | 0.014+0.392 | 0.012+0.144 | | ¬C | 0.126+0.168 | 0.096+0.048 |   So, the Probability is 0.126+0.168=**0.294** |

1. Probability distribution **P**(B) – you can use your answer to part a to compute the answer to this question. **[2 pts]**

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| **Your solution:** |
| |  |  |  | | --- | --- | --- | | Variable | B | ¬B | | C | 0.014+0.392 | 0.012+0.144 | | ¬C | 0.126+0.168 | 0.096+0.048 |   From the Table P(B)= 0.014+0.392+0.126+0.168=**0.7** |

1. Probability distribution **P**(B | ¬C) – you can use your answers to parts a and b to compute the answer to this question. **[2 pts]**

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| **Your solution:** |
| P(B|¬C)=P(B∧¬C)/P(B)  From a) and b)  P(B|¬C)=0.294/0.7=**0.42** |

1. Probability distribution **P**(A, B | ¬C) – you can use your answers from previous parts if they are relevant. **[2 pts]**

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| **Your solution:** |
| **P**(A, B | ¬C)=P(A,B,¬C)/P(¬C)  = P(A∧B∧¬C)/P(¬C)   |  |  |  | | --- | --- | --- | | Variable | C | ¬C | | A,B | 0.014 | 0.126 | | ¬( A,B) | 0.144 | 0.096 | |  |  | P(¬C)=0.222 |   From the table P(A∧B∧¬C)=0.126 ,P(¬C)=0.222  **P**(A, B | ¬C)=0.126/0.222=**0.5675** |